

On Estimation Error Outage for Scalar Gauss-Markov Signals Sent Over Fading Channels

Reza Parseh and Kimmo Kansanen

Department of Electronics and Telecommunications

Norwegian University of Science and Technology

Email: {reza.parseh, kimmo.kansanen}@iet.ntnu.no

Phone: (+47) 73593134, 73594417

Abstract

Measurements of a scalar linear Gauss-Markov process are sent over a fading channel. The fading channel is modeled as independent and identically distributed complex normal random variables with known realization at the receiver. The optimal estimator at the receiver is the Kalman filter. In contrast to the classical Kalman filter theory, given a random channel, the Kalman gain and the error covariance become random. Then the probability distribution function of expected estimation error and its outage probability can be chosen for estimation quality assessment. In this paper and in order to get the estimation error outage, we provide means to characterize the stationary probability distribution function of the random expected estimation error. For the particular case of the i.i.d Rayleigh fading channels, upper and lower bounds for the outage probability are derived which provide insight and simpler means for design purposes. For the high SNR regime, we show that the bounds are tight. We then show that in that case, the outage probability decreases linearly with the inverse of the average channel SNR.

Index Terms

Estimation over Fading Channels, Kalman Filter, Outage Probability

I. INTRODUCTION

Low or zero delay transmission of measurements of a dynamic system to a remote controller/observer is important in applications such as network monitoring and control, wireless sensor networks, and generally in real-time signal processing when the observed signals should be sent over a communication channel. Due to tight delay conditions in many cases, high-performance block-wise coding schemes which incur unacceptable delay, should be avoided. For wireless fading channels, it is possible to send the measurements directly over the channel using uncoded transmission and then perform estimation on the channel outputs at the receiver. Analysis of the signal estimation quality is therefore necessary to ensure satisfactory performance for such applications.

The literature for network communication and control and wireless sensor networks is diverse and rich (c.f. [1]–[4] and the references within). For various applications where the dynamic system follows a Gauss-Markov model and the channel realization is independent of the randomness of the dynamic system, the optimal estimator is the Kalman filter ([5]–[12]). Due to the randomness of the fading channel, the Kalman filter is random and does not necessarily converge to a constant value. The instantaneous estimation error covariance is random as well. It is known that the estimation error covariance matrices develop through a second-order Riccati equation and with the channel matrices being random, the error covariance matrices then constitute a well-known stochastic process referred to as the random Riccati equation (RRE) [13].

In [14], stability of RRE is studied and it is shown that under mild assumptions on the random observability Gramian matrix, it is both L_r and exponentially stable. In [15], the peak covariance stability of the RRE resulting from Kalman filtering with random observation losses is studied. Boundedness of the covariance matrix in the usual sense is also considered in the same work. In [16], an adaptive filtering scheme based on the Riccati equation is proposed for state estimation in network control systems subject to delays, packet drops and missing measurements. In [17], it was shown that sequence of random covariance matrices converges in probability when observations are sent over a packet erasure channel where the erasure event is a Bernoulli i.i.d process. The stationary distributions for infinitely large random matrices with good approximations for dimensions around 10-20 were also found in [18] and [19] for two classes of random Riccati and Lyapunov equations. Also in [20] bounds on the mean of the instantaneous covariance matrices in the RRE formulation are obtained.

In this paper, we study the case when measurements of a scalar Gauss-Markov process are sent over a fading channel with i.i.d. channel realizations. This model best suits e.g. low-cost sensor networks with processing at the fusion center. The samples are sent over the channel using uncoded (also known as analog [21]) transmission after they are obtained, to avoid block coding and the consequent delay. It is assumed that the channel realization is known at the receiver at the time of the observation. The optimum MMSE filter, i.e. the Kalman filter is then random and the exact value of the instantaneous estimation error variance (IEV) cannot be obtained in advance. For that reason, we are in particular interested in statistical characterization of the resulting error.

In the spirit of outage in fading channels, we utilize estimation error outage as a criterion for estimation performance assessment. A similar property, namely distortion outage was proposed in [22] for MIMO block fading channels and as in our case, outage measures are most useful when delay is of concern. In this cases, outage probability provides a more immediate measure of estimation quality for each channel realization compared with more traditional measured, average estimation error. Outage is defined as the event where the IEV exceeds a certain threshold. From a more practical viewpoint, this measure could be used a design parameter for a control or monitoring system which observes the process. We try to find the outage probability and find out how it is related to average channel SNR under certain channel statistics. We show that in the scalar case, the outage measure takes a simple form for high SNR regime, which we believe is insightful for design purposes and further development.

We show that for any i.i.d. fading channel, the first order probability distribution maybe obtained through a recursive integral equation. We then provide upper and lower bounds for the the outage probability for i.i.d. Rayleigh

fading channels and show that the bounds are tight for the high SNR regime. We also show that the outage probability decreases linearly with inverse of the channel SNR in the high SNR regime. This could for instance be used as a rule of thumb method for estimation quality assessment under settings discussed in this paper.

II. SYSTEM MODEL AND PROBLEM DEFINITION

Consider the following scalar complex Gauss-Markov model.

$$\begin{aligned} x(n+1) &= \rho x(n) + u(n), \quad n \geq 1, \quad x(0) \sim \mathcal{CN}(0, M(0)) \\ y(n) &= h(n)x(n) + v(n) \end{aligned} \quad (1)$$

with $u(n)$ and $v(n)$ are white circularly symmetric complex Gaussian random variables with variances σ_u^2 and σ_v^2 , respectively. Consider $h(n)$ to be a circularly symmetric complex Gaussian random variable. This signal model characterizes e.g. measurements of a first-order Gauss-Markov process sent over a fading channel. All the computations are performed in discrete time, and the communication method is analog. The channel is also assumed known at the receiver, but a realization of the random variable with known probability density function. The objective at the receiver is optimal estimation of the signal $x(n)$, given the channel outputs.

Given the previous assumptions, and with $h(n)$ independent from $u(n)$ and $v(n)$, the optimal MMSE estimator of $x(n)$ based on the observations $y(n)$ is the well-known Kalman filter [23] with the following steps, which are

$$\hat{x}(n|n-1) = \rho \hat{x}(n-1|n-1) \quad (2)$$

$$M(n|n-1) = \rho^2 M(n-1|n-1) + \sigma_u^2 \quad (3)$$

$$K(n) = M(n|n-1)h^*(n)[\sigma_v^2 + |h(n)|^2 M(n|n-1)]^{-1} \quad (4)$$

$$\hat{x}(n|n) = \hat{x}(n|n-1) + K(n)(y(n) - h(n)\hat{x}(n|n-1)) \quad (5)$$

$$M(n|n) = (I - K(n)h(n))M(n|n-1) \quad (6)$$

Concisely stated, eq. (2) is the prediction of the current state based on the previous estimated state (*a priori estimate*) using the system model and eq. (3) is the instantaneous expected prediction error. Equation (4) is the corresponding Kalman gain equation and eq. (5) is the correction equation based on the Kalman gain update (*a posteriori estimate*). Finally eq. (6) provides us with the instantaneous estimation error variance.

It is straightforward to show that both the $M(n|n-1)$, i.e. the prediction error and $M(n|n)$, i.e. the estimation error may be written recursively in terms of their previous values and current value of $h(n)$, where one is a deterministic function of the other. The statistical properties of the one may then be acquired using the statistical properties of the other. In the rest of this paper, we study $M(n|n)$ and with some abuse of notation simply call it $M(n)$.

The recursion for $M(n)$ is obtained as follows

$$\begin{aligned}
M(n) &= M(n|n) \\
&= (I - K(n)h(n))M(n|n-1) \\
&= \left(I - \frac{M(n|n-1)|h(n)|^2}{\sigma_v^2 + h^2(n)M(n|n-1)} \right) M(n|n-1) \\
&= M(n|n-1) - \frac{M^2(n|n-1)|h(n)|^2}{\sigma_v^2 + h^2(n)M(n|n-1)} \\
&= \frac{M(n|n-1)\sigma_v^2}{\sigma_v^2 + h^2(n)M(n|n-1)} \\
&= \frac{M(n|n-1)}{1 + h^2(n)M(n|n-1)/\sigma_v^2} \\
&= \frac{\rho^2 M(n-1|n-1) + \sigma_u^2}{1 + \gamma(n)(\rho^2 M(n-1|n-1) + \sigma_u^2)} \\
&= \frac{\rho^2 M(n-1) + \sigma_u^2}{1 + \gamma(n)(\rho^2 M(n-1) + \sigma_u^2)}
\end{aligned} \tag{7}$$

where $\gamma(n) = |h(n)|^2/\sigma_v^2$ corresponds to the instantaneous channel quality.

In order to characterize the random estimation outage event, we define estimation error outage probability (EOP) as

$$P_{\text{out}}^n(M_{\text{th}}) = \Pr(M(n) \geq M_{\text{th}}) \tag{8}$$

and in particular the asymptotic EOP which is of interest, in order to characterize the steady-state distributions, i.e.

$$P_{\text{out}}(M_{\text{th}}) = \lim_{n \rightarrow \infty} P_{\text{out}}^n(M_{\text{th}}) = \lim_{n \rightarrow \infty} \Pr(M(n) \geq M_{\text{th}}) \tag{9}$$

Clearly $P_{\text{out}}^n(M_{\text{th}}) = 1 - F_{M(n)}(M_{\text{th}})$ and $P_{\text{out}}(M_{\text{th}}) = 1 - F_M(M_{\text{th}})$, where $F_{M(n)}(M)$ ($F_M(M)$) is the (asymptotic) cumulative distribution function of $M(n)$.

III. STATISTICAL PROPERTIES OF INSTANTANEOUS ESTIMATION ERROR VARIANCE

In this section we study the asymptotic probability distribution function of the IEV. Because $F_M(M)$ and $f_M(M)$ are related with a linear operation (derivative), we begin to study $f_M(M)$. In that way, not only the EOP will readily be obtained with one integration, other moments such as mean and variance, if needed, can be obtained or bounded.

A. Asymptotic pdf of the instantaneous estimation error variance

Given (7), it is easy to verify that for any arbitray positive real number M , $M(n) \leq M$ leads to

$$\gamma(n) \geq \frac{1}{M} - \frac{1}{\rho^2 M(n-1) + \sigma_u^2}.$$

Also, we have that $\gamma(n) \geq 0$ and $0 < M(n) < M_{\text{max}}$, where M_{max} , the upper limit for $M(n)$ is obtained from

$$M_{\text{max}} = \begin{cases} \infty, & |\rho| \geq 1 \\ \frac{\sigma_u^2}{1-\rho^2}, & |\rho| < 1. \end{cases} \tag{10}$$

M_{\max} is effectively the estimation error variance for the worst channel, i.e. $\gamma(n) = 0$ with probability 1. In that case, the best estimator is the average mean, i.e. $\hat{x}(n) = E(x(n)) = 0$ and therefore the estimation error variance is equal to M_{\max} . Also note that the case $|\rho| \geq 1$ is of little practical importance in our case, because for a divergent signal, analog transmission would not be practical. It is however included to show that the analysis does not depend on the value of ρ .

Given the above limits and conditions for $\gamma(n)$ and $M(n)$ and according to [24], it is possible to get the cdf of $M(n)$, i.e. $F_{M(n)}(M)$ as follows.

$$F_{M(n)}(M) = \int_0^{M_{\max}} \int_{\frac{1}{M} - \frac{1}{\rho^2 M(n-1) + \sigma_u^2}}^{\infty} f_{\gamma(n), M(n-1)}(\gamma(n), M(n-1)) d\gamma(n) dM(n-1) \quad (11)$$

where $f_{\gamma(n), M(n-1)}(\gamma(n), M(n-1))$ is the joint pdf of $\gamma(n)$ and $M(n-1)$. The pdf for $M(n)$ is then obtained by simply applying $f_{M(n)}(M) = \frac{\partial}{\partial M} F_{M(n)}(M)$. That leads to

$$\begin{aligned} f_{M(n)}(M) &= \int_0^{M_{\max}} \frac{\partial}{\partial M} \int_{\frac{1}{M} - \frac{1}{\rho^2 M(n-1) + \sigma_u^2}}^{\infty} f_{\gamma(n), M(n-1)}(\gamma(n), M(n-1)) d\gamma(n) dM(n-1) \\ &= \int_0^{M_{\max}} \frac{1}{M^2} f_{\gamma(n), M(n-1)} \left(\frac{1}{M} - \frac{1}{\rho^2 M(n-1) + \sigma_u^2}, M(n-1) \right) dM(n-1) \end{aligned} \quad (12)$$

or with some change of notation,

$$f_{M(n)}(M) = \int_0^{M_{\max}} \frac{1}{M^2} f_{\gamma(n), M(n-1)} \left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}, m \right) dm \quad (13)$$

Now if we assume that $\gamma(n)$ is independent of $M(n-1)$ ($\gamma(n) \perp\!\!\!\perp M(n-1)$), we may rewrite (13) as

$$f_{M(n)}(M) = \frac{1}{M^2} \int_0^{M_{\max}} f_{\gamma(n)} \left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2} \right) f_{M(n-1)}(m) dm \quad (14)$$

Note that as we have assumed i.i.d. channels, then we have that $\gamma(n) \perp\!\!\!\perp \gamma(i)$ for $i < n$. We can simply have that $\gamma(i) \perp\!\!\!\perp M(0)$ ($M(0)$ is a constant). As a result, we see that $\gamma(n) \perp\!\!\!\perp M(n-1)$ because $M(n-1)$ is a function of $M(0)$ and $\gamma(1), \gamma(2), \dots, \gamma(n-1)$ only. Thus i.i.d. channel assumption is a sufficient condition to get the main result in (14).

Finally,

$$\lim_{n \rightarrow \infty} f_{M(n)}(M) = \lim_{n \rightarrow \infty} f_{M(n-1)}(M) = f_M(M). \quad (15)$$

As a result the steady-state version of (14) can be rewritten as

$$f_M(M) = \frac{1}{M^2} \int_0^{M_{\max}} f_{\gamma} \left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2} \right) f_M(m) dm \quad (16)$$

Note that the transition from $f_{\gamma(n)}()$ to $f_{\gamma}()$ is possible because the channel samples are from an i.i.d. distribution. To be more specific with (16), we use the fact that $\gamma(n) \geq 0$. That necessitates that the argument of the function $f_{\gamma}()$ should always be positive. Clearly, if $M \leq \sigma_u^2$, the term $\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2}$ is always positive. However for $M > \sigma_u^2$, the integral should be taken over the range of γ where $\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2} \geq 0$, i.e. for $m \geq \frac{M - \sigma_u^2}{\rho^2}$. With this background, we can finally provide the following lemma that describes the asymptotic pdf of $M(n)$, i.e. $f_M(M)$ in terms of

itself integrated with a kernel which is a function of the instantaneous channel SNR. Solving this equation leads to $f_M(M)$ and with one integration to P_{out} , which is the target.

Lemma 1: Asymptotic pdf of $M(n)$, i.e. $f_M(M)$ can be obtained from the following equation

$$f_M(M) = \begin{cases} \frac{1}{M^2} \int_0^{M_{\max}} f_\gamma \left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2} \right) f_M(m) dm, & M \leq \sigma_u^2 \\ \frac{1}{M^2} \int_{\frac{M - \sigma_u^2}{\rho^2}}^{M_{\max}} f_\gamma \left(\frac{1}{M} - \frac{1}{\rho^2 m + \sigma_u^2} \right) f_M(m) dm, & M > \sigma_u^2 \end{cases} \quad (17)$$

The solution is general and is explicitly given in terms of instantaneous channel SNR pdf and system parameters. Though (17) can easily be solved numerically, the general closed-form solution does not seem to be readily attainable. In the following, we have focused on the important case of Rayleigh fading channels where $f_\gamma(\gamma) = \lambda e^{-\lambda\gamma} \mathcal{U}(\gamma)$ ($\mathcal{U}()$ is the unit step function). Note that with this definition, $\lambda = 1/E(\gamma(n)) = \sigma_v^2/E(|h(n)|^2)$, i.e. stronger channels yield smaller values for λ and vice versa.

B. Pdf of the instantaneous estimation error variance under Rayleigh fading channel

We can rewrite (17) given that channel is i.i.d. Rayleigh fading. Using that we obtain

$$f_M(M) = \frac{\lambda}{M^2} \exp\left(\frac{-\lambda}{M}\right) \begin{cases} \int_0^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm, & M \leq \sigma_u^2 \\ \int_{\frac{M - \sigma_u^2}{\rho^2}}^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm, & M > \sigma_u^2 \end{cases} \quad (18)$$

which in order to get more insight and with some algebraic manipulation, can also be written as

$$f_M(M) = \frac{\lambda}{M^2} \exp\left(\frac{-\lambda}{M}\right) \begin{cases} \kappa, & M \leq \sigma_u^2 \\ \kappa - \int_0^{\frac{M - \sigma_u^2}{\rho^2}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm, & M > \sigma_u^2 \end{cases} \quad (19)$$

where

$$\kappa = \int_0^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm. \quad (20)$$

Though in general κ depends on the pdf itself, (19) is insightful in the sense that it shows the exact shape of the pdf for the first part where $M \leq \sigma_u^2$.

Typical shapes of such pdf's which are obtained through Monte-Carlo simulations are depicted in Fig. 1 to further highlight the points mentioned. For Fig. 1, it is assumed that $\sigma_u^2 = \sigma_v^2 = 1$, $\lambda = 1, 0.5, 0.25$ (SNR = 0, 3, 6 dB respectively), and $\rho = 0.95$. Note that the pdf support is theoretically bounded in this case at point $M_{\max} = \sigma_u^2/(1 - \rho^2) \approx 10.26$ (not shown in the figure due to its insignificance). Also note that the break point, $M = \sigma_u^2$ where the pdf changes shape is quite visible in Fig. 1. Also, from (19), it is easily verified that the pdf has an extremum point at $M = \lambda/2$ for the given SNR values. This extremum point is only a function of the average channel SNR, i.e. $E(\gamma(n)) = 1/\lambda$.

The break point $M = \sigma_u^2$ corresponds to the steady-state variance of the signal when there is no correlation ($\rho = 0$), whereas the point $M_{\max} = \sigma_u^2/(1 - \rho^2)$ corresponds to the upper limit for the support of $F_M(M)$ (maximum value for the IEV) for the worst channel ($\gamma(n) = 0$) when no information gets passed the channel and the estimator is equal to $\hat{x}(n) = E(x(n)) = 0$. It is quite visible and theoretically verifiable that the pdf tail

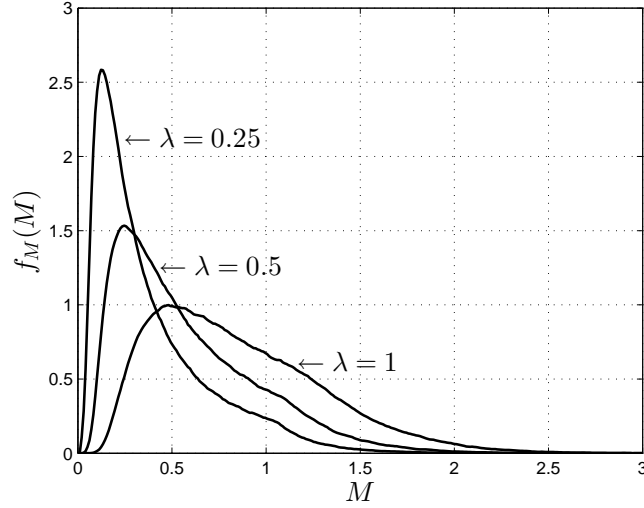


Fig. 1. Asymptotic pdf of $M(n)$ for $\sigma_u^2 = \sigma_v^2 = 1$, $\rho = 0.95$, $\lambda = 1, 0.5, 0.25$ (SNR = 0, 3, 6 dB respectively). Note the break point at $M = \sigma_u^2$.

vanishes rapidly after the break point. Also that the higher the threshold, the lower the outage probability would be. As a result, getting bounds on the first part helps with understanding the pdf behavior better and at the same time get approximate values and bounds for $P_{\text{out}}(M_{\text{th}})$. Using (19) and (20), we find upper and lower bounds for κ , approximations for the pdf and upper and lower bounds for the outage for the first part ($M \leq \sigma_u^2$). Another insight from (20) is that the pdf shape is independent of whether the system is stable ($\rho < 1$) or unstable ($\rho \geq 1$), though the value of κ depends on ρ .

Though the pdf is given by the equation $F_M(M) = \frac{\kappa\lambda}{M^2} \exp(-\frac{\lambda}{M})$ ($M \leq \sigma_u^2$), the exact value of κ depends on the whole pdf and cannot be known without solving (19). However, it is possible to obtain the following bounds for κ , namely $\kappa_l < \kappa < \kappa_u$, which later on are used to characterize two functions $P_{\text{out}}^l(M_{\text{th}})$ and $P_{\text{out}}^u(M_{\text{th}})$ for which $P_{\text{out}}^l(M_{\text{th}}) < P_{\text{out}}(M_{\text{th}}) < P_{\text{out}}^u(M_{\text{th}})$ for all $M \leq \sigma_u^2$.

Lemma 2: For all $M \leq \sigma_u^2$, we have $\kappa_l < \kappa < \kappa_u$, where

$$\kappa_u = \frac{1}{\left(a_\kappa \exp\left(\frac{-\lambda}{\sigma_u^2(1+\rho^2)}\right) + \exp\left(-\frac{\lambda}{\sigma_u^2}\right)\right)} \quad (21)$$

$$\kappa_l = \frac{1}{\left(a_\kappa \exp\left(\frac{-\lambda}{\rho^2 M_{\text{max}} + \sigma_u^2}\right) + \exp\left(-\frac{\lambda}{\sigma_u^2}\right)\right)} \quad (22)$$

where we have defined

$$a_\kappa = 1 - \int_0^{\sigma_u^2} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) \left(\frac{\lambda}{m^2}\right) \exp\left(-\frac{\lambda}{m}\right) dm \quad (23)$$

Proof: See Appendix A.

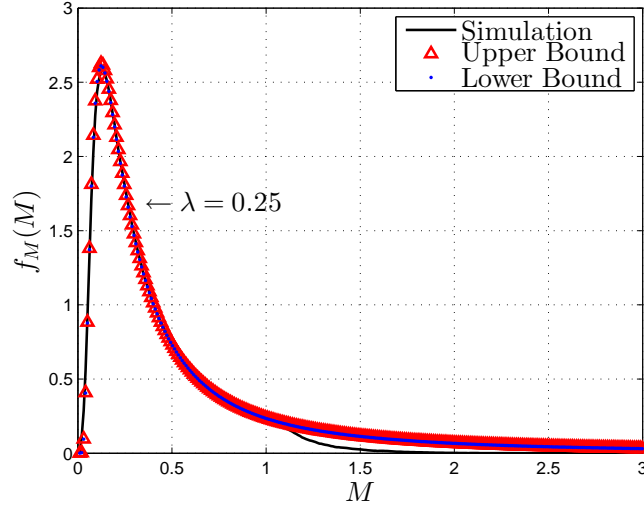


Fig. 2. Asymptotic pdf of $M(n)$ and its approximates using upper and lower bounds for κ given that $\sigma_u^2 = \sigma_v^2 = 1$, $\rho = 0.95$, $\lambda = 0.25$ (SNR = 6 dB).

Note that for stable systems, $M_{\max} = \frac{\sigma_u^2}{1-\rho^2}$ and not surprisingly $\rho^2 M_{\max} + \sigma_u^2 = M_{\max}$. Then we get

$$\kappa_l^b = \frac{1}{\left(a_\kappa \exp\left(\frac{-\lambda}{M_{\max}}\right) + \exp\left(-\frac{\lambda}{\sigma_u^2}\right)\right)} \quad (24)$$

For unstable systems we have $M_{\max} \rightarrow \infty$, and as a result

$$\kappa_l^\infty = \frac{1}{\left(a_\kappa + \exp\left(-\frac{\lambda}{\sigma_u^2}\right)\right)} \quad (25)$$

To show how tight the bounds are, we have plotted the simulated pdf and two approximates using the bounds for κ in Fig. 2, given that $\sigma_u^2 = \sigma_v^2 = 1$, $\lambda = 0.25$ (SNR = 6dB), and $\rho = 0.95$.

With Lemma 2 at hand, we are now ready to present the bounds for $P_{\text{out}}(M_{\text{th}})$. We then show that the bounds are tight for the high average SNR regime, i.e. $\lambda \rightarrow 0$. This is discussed in the next section.

IV. BOUNDS ON OUTAGE PROBABILITY FOR HIGH SNR

In this section we get upper and lower bounds for $P_{\text{out}}(M_{\text{th}})$. We show that for a given non-zero M_{th} , EOP decreases with inverse of average channel SNR. We then discuss the conditions where the bounds are tight, which happens for the high SNR regime.

As defined before, $P_{\text{out}}(M_{\text{th}})$ is given by

$$P_{\text{out}}(M_{\text{th}}) = \int_{M_{\text{th}}}^{M_{\max}} F_M(M) dM \quad (26)$$

For $M \leq \sigma_u^2$, we get

$$P_{\text{out}}(M_{\text{th}}) = \int_{M_{\text{th}}}^{M_{\max}} \frac{\kappa \lambda}{M^2} \exp\left(\frac{-\lambda}{M}\right) dM = 1 - \kappa \exp\left(\frac{-\lambda}{M_{\text{th}}}\right) \quad (27)$$

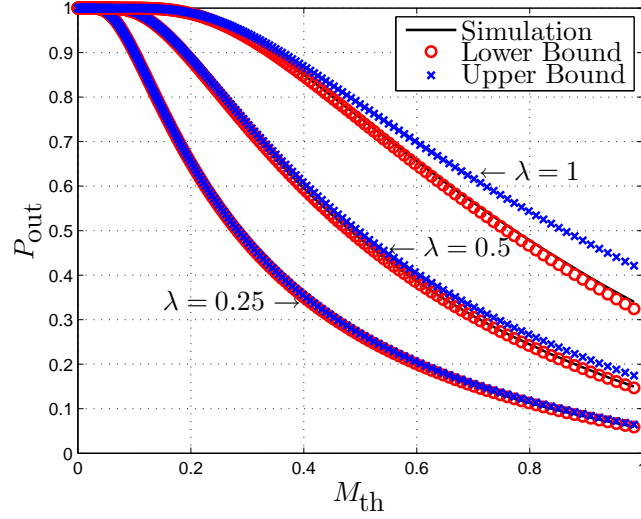


Fig. 3. $P_{\text{out}}(M_{\text{th}})$ and its upper and lower bounds for $\sigma_u^2 = \sigma_v^2 = 1$, $\rho = 0.95$, $\lambda = 1, 0.5, 0.25$ (SNR = 0, 3, 6 dB).

As shown in the previous section, $\kappa_l < \kappa < \kappa_u$. As a result, we get

$$1 - \kappa_u \exp\left(\frac{-\lambda}{M_{\text{th}}}\right) < P_{\text{out}}(M_{\text{th}}) < 1 - \kappa_l \exp\left(\frac{-\lambda}{M_{\text{th}}}\right) \quad (28)$$

which give us an upper bound and a lower bound for $P_{\text{out}}(M_{\text{th}})$. Figure 3 depicts the outage probability and the bounds for the case when $\sigma_u^2 = \sigma_v^2 = 1$, $\lambda = 1, 0.5, 0.25$, and $\rho = 0.95$ and for $M \leq \sigma_u^2$.

As seen in Fig. 3, a smaller λ yields a smaller outage probability. It is interesting to see how the increase in the SNR, i.e. a decrease in the value of λ , will lead to a lower outage probability. Also, we will show that the bounds are tight for high SNR, i.e. $\lambda \rightarrow 0$.

Lemma 3: The upper and lower bounds for $P_{\text{out}}(M_{\text{th}})$ are tight for $\lambda \rightarrow 0$.

Proof: As shown in Appendix B, we have that

$$\lim_{\lambda \rightarrow 0} \kappa = \lim_{\lambda \rightarrow 0} \kappa_u = \lim_{\lambda \rightarrow 0} \kappa_l = 1, \quad (29)$$

which proves the lemma.

It is also interesting to see how fast κ converges to 1 for small values of λ and for which values of λ , the upper and lower bound are approximately equal. This is depicted in Fig. 4. Quite visibly, for values of λ smaller than 0.01 (SNR greater than 20dB) the upper and lower bounds for κ are very close. Due to the fact that the bounds for κ are tight, the bounds for $P_{\text{out}}(M_{\text{th}})$ are also tight. Even for the range of medium SNR depicted in Fig. 3, it is quite visible that the upper and lower bounds for the outage probability are quite close to the one obtained from the simulation and that increasing the SNR improves their accuracy. It is also quite visible from Fig. 4 that the linear approximation obtained from the Taylor series expansion of κ as a function of λ (see Appendix B) is accurate.

At this point we are ready to present the asymptotic behavior of the outage probability for the high SNR regime. This is presented in Lemma 4.

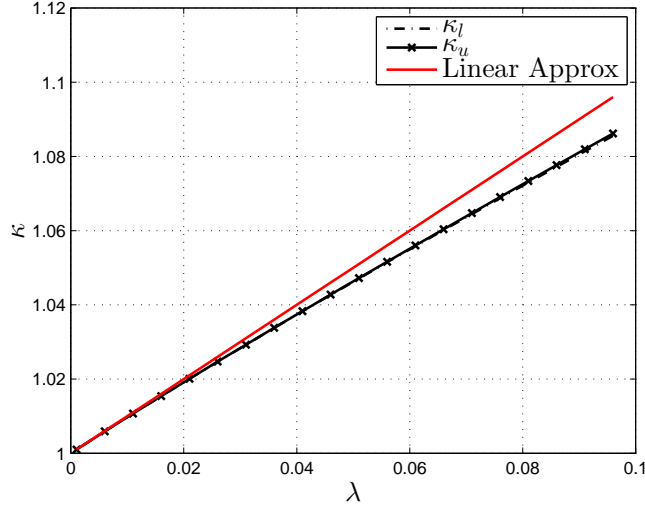


Fig. 4. κ_l, κ_u as a function of λ for $\sigma_u^2 = \sigma_v^2 = 1$, $\rho = 0.95$.

Lemma 4: For the high SNR regime, $P_{\text{out}}(M_{\text{th}})$ decreases approximately linearly with λ .

Proof: We can use the Taylor series expansion of κ around $\lambda = 0$ from Appendix B to approximate the outage probability for the high SNR regime. Using the Taylor series expansion for κ (from App. B) and $\exp(\frac{-\lambda}{M_{\text{th}}})$, we obtain

$$\begin{aligned}
 P_{\text{out}}(M_{\text{th}}) &= 1 - \kappa \exp\left(\frac{-\lambda}{M_{\text{th}}}\right) \\
 &= 1 - \left(1 + \frac{\lambda}{\sigma_u^2} + \mathcal{O}(\lambda^2)\right) \left(1 - \frac{\lambda}{M_{\text{th}}} + \mathcal{O}(\lambda^2)\right) \\
 &= \left(\frac{1}{M_{\text{th}}} - \frac{1}{\sigma_u^2}\right) \lambda + \mathcal{O}(\lambda^2)
 \end{aligned} \tag{30}$$

For small λ , $\mathcal{O}(\lambda^2)$ vanishes faster than λ and as a result we could claim $P_{\text{out}}(M_{\text{th}})$ is approximately a linear function of λ . The linear approximation is depicted in Fig. 5 for $\lambda \in [0.001, 0.01]$. The consequence of this linearity is that though increasing the channel SNR helps with outage probability, it does not help significantly and it may be beneficial to find a tradeoff between power consumption and required outage probability for the application at hand.

V. CONCLUSIONS

In this paper, a recursive integral equation approach was presented for finding the pdf of the instantaneous estimation error variance for MMSE estimation of scalar Gauss-Markov signals sent over fading channels. We also utilized the notion of estimation error outage as a means of characterizing the estimation accuracy. It was shown that the pdf can be written as a two-part function over the domain of instantaneous estimation error variance values. The first part of the pdf, i.e. the range up to the Gauss-Markov process variance also corresponding to higher outage

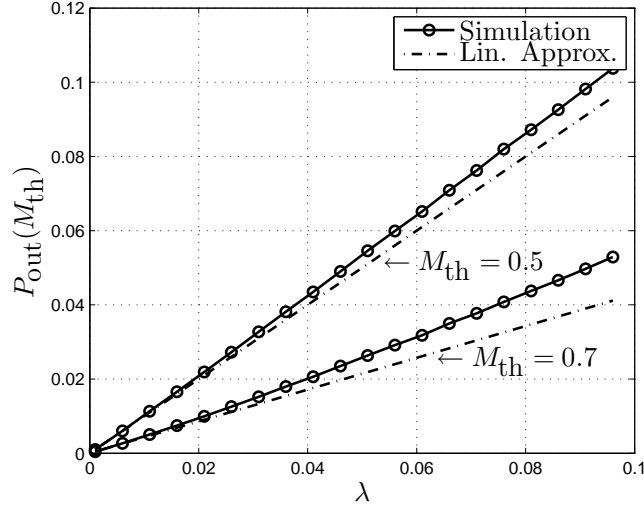


Fig. 5. P_{out} as a function of λ for $\sigma_u^2 = \sigma_v^2 = 1$, $\rho = 0.95$ and its linear approximation

probabilities, follows a closed-form non-recursive equation. As a result and for the case of i.i.d. Rayleigh fading channels, the outage probability can be approximated with a closed-form formula for the first part. Upper and lower bounds on the estimation error outage probability were also obtained to simplify characterization of estimation error outage. The presented bounds become were shown to be tight when the SNR grows unbounded. In the end, it was shown that the outage decreases linearly with inverse of the SNR for the high SNR regime.

APPENDIX A

UPPER AND LOWER BOUNDS FOR κ

We begin to rewrite $F_M(M)$ in the following manner for simplicity,

$$f_M(M) = \begin{cases} \frac{\kappa\lambda}{M^2} \exp\left(\frac{-\lambda}{M}\right) & M \leq \sigma_u^2 \\ g(M) & M > \sigma_u^2 \end{cases} \quad (31)$$

We have that $F_M(M)$ is a pdf, therefore $\int_0^{M_{\max}} F_M(M) dM = 1$. As a result,

$$\begin{aligned} 1 &= \int_0^{M_{\max}} F_M(M) dM = \int_0^{\sigma_u^2} \frac{\kappa\lambda}{M^2} \exp\left(\frac{-\lambda}{M}\right) + \int_{\sigma_u^2}^{M_{\max}} g(M) dM \\ &= \kappa \exp\left(\frac{-\lambda}{M}\right) \Big|_0^{\sigma_u^2} + \int_{\sigma_u^2}^{M_{\max}} g(M) dM \\ &= \kappa \exp\left(\frac{-\lambda}{\sigma_u^2}\right) + \int_{\sigma_u^2}^{M_{\max}} g(M) dM \end{aligned} \quad (32)$$

which gives

$$\int_{\sigma_u^2}^{M_{\max}} g(M) dM = 1 - \kappa \exp\left(\frac{-\lambda}{\sigma_u^2}\right) \quad (33)$$

Now let's take

$$\sigma_u^2 < m < M_{\max}.$$

Then we have,

$$(\rho^2 + 1)\sigma_u^2 < \rho^2 m + \sigma_u^2 < \rho^2 M_{\max} + \sigma_u^2,$$

and

$$\exp\left(\frac{\lambda}{\rho^2 M_{\max} + \sigma_u^2}\right) < \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) < \exp\left(\frac{\lambda}{(\rho^2 + 1)\sigma_u^2}\right). \quad (34)$$

Now we have that

$$\int_{\sigma_u^2}^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) g(m) dm > \int_{\sigma_u^2}^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 M_{\max} + \sigma_u^2}\right) g(m) dm \quad (35)$$

$$\int_{\sigma_u^2}^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) g(m) dm < \int_{\sigma_u^2}^{M_{\max}} \exp\left(\frac{\lambda}{(\rho^2 + 1)\sigma_u^2}\right) g(m) dm \quad (36)$$

Next if we note that for $M \leq \sigma_u^2$, we obtain

$$\begin{aligned} f(M) &= \frac{\lambda}{M^2} \exp\left(\frac{-\lambda}{M}\right) \int_0^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm \\ &= \frac{\kappa \lambda}{M^2} \exp\left(\frac{-\lambda}{M}\right) \end{aligned} \quad (37)$$

$$\begin{aligned} &= \frac{\lambda}{M^2} \exp\left(\frac{-\lambda}{M}\right) \left(\int_0^{\sigma_u^2} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) \left(\frac{\kappa \lambda}{m^2}\right) \exp\left(\frac{-\lambda}{m}\right) dm \right. \\ &\quad \left. + \int_{\sigma_u^2}^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) g(m) dm \right) \end{aligned} \quad (38)$$

From which by equating (37) and (38) and removing common terms on both sides, we deduce that

$$\begin{aligned} \kappa &= \kappa \int_0^{\sigma_u^2} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) \left(\frac{\lambda}{m^2}\right) \exp\left(\frac{-\lambda}{m}\right) dm \\ &\quad + \int_{\sigma_u^2}^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) g(m) dm \end{aligned} \quad (39)$$

And then,

$$\kappa = \frac{\int_{\sigma_u^2}^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) g(m) dm}{1 - \int_0^{\sigma_u^2} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) \left(\frac{\lambda}{m^2}\right) \exp\left(\frac{-\lambda}{m}\right) dm} \quad (40)$$

Now by letting

$$a_\kappa = 1 - \int_0^{\sigma_u^2} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) \left(\frac{\lambda}{m^2}\right) \exp\left(\frac{-\lambda}{m}\right) dm \quad (41)$$

and integrating (35) into (40) while using (33), we get

$$\begin{aligned} \kappa a_\kappa &> \int_{\sigma_u^2}^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 M_{\max} + \sigma_u^2}\right) g(m) dm \\ &> \exp\left(\frac{\lambda}{\rho^2 M_{\max} + \sigma_u^2}\right) \int_{\sigma_u^2}^{M_{\max}} g(m) dm \end{aligned} \quad (42)$$

$$> \exp\left(\frac{\lambda}{\rho^2 M_{\max} + \sigma_u^2}\right) (1 - \kappa \exp\left(\frac{-\lambda}{\sigma_u^2}\right)) \quad (43)$$

which leads to

$$\kappa > \frac{1}{(a_\kappa \exp(\frac{-\lambda}{\rho^2 M_{\max} + \sigma_u^2}) + \exp(\frac{-\lambda}{\sigma_u^2}))} \quad (44)$$

So we finally get

$$\kappa_l = \frac{1}{(a_\kappa \exp(\frac{-\lambda}{\rho^2 M_{\max} + \sigma_u^2}) + \exp(\frac{-\lambda}{\sigma_u^2}))} \quad (45)$$

The same procedure also holds for κ_u by integrating (36) into (20) while using (33). We then get

$$\kappa_u = \frac{1}{(a_\kappa \exp(\frac{-\lambda}{\sigma_u^2(1+\rho^2)}) + \exp(-\frac{\lambda}{\sigma_u^2}))} \quad (46)$$

APPENDIX B

HIGH SNR LIMITS FOR κ , κ_u , κ_l

In this section we show that κ , κ_u , κ_l converge to 1 in the high SNR regime, i.e. in the limit of $\lambda \rightarrow 0$. We also get second term of the Taylor series expansion for κ to accommodate for the high SNR linear approximation for the outage probability in Lemma 4.

We have

$$\kappa_u = \frac{1}{(a_\kappa \exp(\frac{-\lambda}{\sigma_u^2(1+\rho^2)}) + \exp(-\frac{\lambda}{\sigma_u^2}))} \quad (47)$$

For finite σ_u^2 , the condition $\lambda \rightarrow 0$ can be extended to $\lambda/\sigma_u^2 \rightarrow 0$. We make this assumption to simplify the calculations. Assume $\lambda = \alpha\sigma_u^2$, then

$$\kappa_u = \frac{1}{(a_\kappa(\alpha) \exp(\frac{-\alpha}{(1+\rho^2)}) + \exp(-\alpha))} \quad (48)$$

For finite σ_u^2 , the condition $\lambda \rightarrow 0$ will be equal to $\alpha \rightarrow 0$. We can then see that

$$\lim_{\alpha \rightarrow 0} \kappa_u(\alpha) = \frac{1}{1 + \lim_{\alpha \rightarrow 0} a_\kappa(\alpha)} \quad (49)$$

In the following we show that $\lim_{\alpha \rightarrow 0} a_\kappa(\alpha) = 0$. We have

$$\begin{aligned} a_\kappa(\alpha) &= 1 - \int_0^{\sigma_u^2} \exp(\frac{\lambda}{\rho^2 m + \sigma_u^2}) \left(\frac{\lambda}{m^2} \right) \exp(\frac{-\lambda}{m}) dm \\ &= 1 - \int_0^1 \exp(\frac{\alpha}{1 + \rho^2 v}) \left(\frac{\alpha}{v^2} \right) \exp(\frac{-\alpha}{v}) dv \end{aligned} \quad (50)$$

where we made the change of variable $v = \frac{m}{\sigma_u^2}$. Now take $a_\kappa(\alpha) = 1 - I(\alpha)$, where

$$I(\alpha) = \int_0^1 \exp\left(\frac{\alpha}{1+\rho^2 v}\right) \left(\frac{\alpha}{v^2}\right) \exp\left(\frac{-\alpha}{v}\right) dv \quad (51)$$

$$\begin{aligned} &= \exp\left(\frac{\alpha}{1+\rho^2 v}\right) \exp\left(\frac{-\alpha}{v}\right) \Big|_0^1 \\ &\quad - \int_0^1 \exp\left(\frac{-\alpha}{v}\right) \exp\left(\frac{\alpha}{1+\rho^2 v}\right) \left(\frac{-\alpha \rho^2}{(1+\rho^2 v)^2}\right) dv \end{aligned} \quad (52)$$

$$\begin{aligned} &= \exp\left(\frac{\alpha}{1+\rho^2}\right) \exp(-\alpha) \\ &\quad + \rho^2 \alpha \int_0^1 \exp\left(\frac{-\alpha}{v}\right) \exp\left(\frac{\alpha}{1+\rho^2 v}\right) \frac{1}{(1+\rho^2 v)^2} dv \end{aligned} \quad (53)$$

Now because all of the functions $\exp(\frac{-\alpha}{v})$, $\exp(\frac{\alpha}{1+\rho^2 v})$, $\frac{1}{(1+\rho^2 v)^2}$ are finite for $v \in [0, 1]$, then the integral term in (53) is also finite. As a result, $\lim_{\alpha \rightarrow 0} I(\alpha) = 1$ and $\lim_{\alpha \rightarrow 0} \kappa_u(\alpha) = 1$. Similar results also hold for $\kappa_l^\infty, \kappa_l^b$.

To get the limiting behavior for κ when $\lambda \rightarrow 0$, we use the original definition for κ , i.e.

$$\kappa = \int_0^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm \quad (54)$$

As a prerequisite for lemmas 3 and 4, we need the Taylor series expansion for κ around the point $\lambda = 0$.

We begin by first showing that the cumulative distribution function of IEV, i.e. $F_M(M)$ approaches the step function when $\lambda \rightarrow 0$. We have that

$$F_M(M) = 1 - P_{\text{out}}(M) = \kappa \exp\left(\frac{-\lambda}{M}\right) \quad (55)$$

Now for any $M > 0$, we have

$$\begin{aligned} \lim_{\lambda \rightarrow 0} F_M(M) &= \lim_{\lambda \rightarrow 0} \kappa \exp\left(\frac{-\lambda}{M}\right) \\ &= \lim_{\lambda \rightarrow 0} \kappa \end{aligned} \quad (56)$$

Now,

$$\begin{aligned} \lim_{\lambda \rightarrow 0} \kappa &= \lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm \\ &= \lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} \sum_{l=0}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 m + \sigma_u^2)^l} f_M(m) dm \\ &= \lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} f_M(m) dm \\ &\quad + \lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 m + \sigma_u^2)^l} f_M(m) dm \\ &= \lim_{\lambda \rightarrow 0} 1 + \lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 m + \sigma_u^2)^l} f_M(m) dm \\ &= 1 + \lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 m + \sigma_u^2)^l} f_M(m) dm \end{aligned} \quad (57)$$

but for $m \geq 0$, we have $\frac{1}{\rho^2 m + \sigma_u^2} \leq \frac{1}{\sigma_u^2}$. As a result

$$\begin{aligned} \lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 m + \sigma_u^2)^l} f_M(m) dm &\leq \\ \lim_{\lambda \rightarrow 0} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\sigma_u^2)^l} \int_0^{M_{\max}} f_M(m) dm & \end{aligned} \quad (58)$$

and thus

$$\begin{aligned} \lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 m + \sigma_u^2)^l} f_M(m) dm &\leq \\ \lim_{\lambda \rightarrow 0} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\sigma_u^2)^l} & \end{aligned} \quad (59)$$

but

$$\lim_{\lambda \rightarrow 0} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\sigma_u^2)^l} = \lim_{\lambda \rightarrow 0} (\exp(\frac{\lambda}{\sigma_u^2}) - 1) = 0 \quad (60)$$

Therefore,

$$\lim_{\lambda \rightarrow 0} \int_0^{M_{\max}} \sum_{l=1}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 m + \sigma_u^2)^l} f_M(m) dm = 0 \quad (61)$$

and finally, $\lim_{\lambda \rightarrow 0} \kappa = 1$. That shows that $F_M(M)$ approached the unit step function when $\lambda \rightarrow 0$ and as a result $F_M(M)$ approaches the Dirac's delta function when $\lambda \rightarrow 0$. With this assumption we have

$$\begin{aligned} \kappa &= \int_0^{M_{\max}} \exp\left(\frac{\lambda}{\rho^2 m + \sigma_u^2}\right) f_M(m) dm \\ &= \int_0^{M_{\max}} \sum_{l=0}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 m + \sigma_u^2)^l} f_M(m) dm \\ &= \int_0^{M_{\max}} \sum_{l=0}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\rho^2 y + \sigma_u^2)^l} f_M(m) dm \\ &= \sum_{l=0}^{\infty} \frac{\lambda^l}{l!} \int_0^{M_{\max}} \frac{1}{(\rho^2 y + \sigma_u^2)^l} f_M(m) dm \\ &= \sum_{l=0}^{\infty} \frac{\lambda^l}{l!} \frac{1}{(\sigma_u^2)^l} \end{aligned} \quad (62)$$

which is the Taylor series expansion for κ around $\lambda = 0$ to be used in lemma 3 and 4.

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